



## INITIAL STATES AND TRANSITIONAL EXPENSES IN PRODUCTION AND TRANSPORT SYSTEMS OPTIMIZATION

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### ABSTRACT

The production and transport systems optimization models, as well as classical transport or distributive problems, mainly concentrate on finding of the plan, optimum according to own characteristics, and don't take into account costs on system transition from its given (initial) state to defined as optimum.

The neglect of such transitional expenses leads to certain losses of the corresponding models' adequacy and to obtaining the absolutely-optimal solution, which is invariant to initial system state.

In our paper we introduce and investigate the notion of relatively-optimal plans.

The basic types of dependences of relatively-optimal plans and maximum achievable profit on the production and transport system initial state under the different functions of transitional expenses are determined.

Corresponding computer calculations are also carried out and confirmed our theoretical results and conclusions.

From the mathematical point of view, the accounting of initial states and the transitional expenses, on the one hand, complicates the corresponding models and computing processes, however on the other hand – does the models even more interesting, creates additional non-trivial effects.

**Keywords:** absolutely and relatively optimal plans, optimization models, profit.

JEL classification: C51, C61, O21

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### INTRODUCTION

The researches of supply chains that are linking producers, transporters and consumers of products, forecasting of their expected behavior are becoming of particularly great value at the present stage.

However, in numerous literature on supply chains, which has been extensively published in recent years (Grazia Speranza, 2018; Zhang et al, 2020), in particular the integrated supply chains (Bowersox & Closs, 1996; Shapiro, 2000; Geunes et al, 2002; Poirier, 2003; Voss & Woodruff, 2003; Stadler & Kilger, 2004; Simchi-Levi et al, 2004; Shaelaie et al, 2018; Guajardo et al, 2018) relevant issues are considered primarily on the descriptive level, the best case scenario with the simplest calculation formulas.

From our point of view, the study of supply chains systems with the use and development of relevant results in microeconomics (Varian, 1992; Jehle & Reny, 2000; Nicholson, 2000; Mathis & Koscianski, 2002)



and the theory of organization of industrial markets (Church & Ware, 2000; Pepall et al, 2001; Schmalensee & Willig, 2002; Carlton & Perloff, 2004) deserves serious attention.

The constructed production and transport systems optimization models (Kholodenko, 2020), as well as classical transport or distributive problems, mainly concentrate on finding of the plan, optimum according to own characteristics, and don't take into account costs on system transition from its given (initial) state to defined as optimum.

The neglect of such transitional expenses leads to certain losses of the corresponding models' adequacy and to obtaining the optimal solution, which is invariant to initial system state. However, this initial state, as a rule, explicitly or implicitly is accepted as identical (zero), i.e. the production and transport systems, which are projected are in question.

## METHODOLOGY

However, in investment projection (Kong et al, 2022), which is similar in meaning, the criteria indicator NPV (net present value) is formed not only as the sum of the present value of future cash flows – but less initial investments, i.e. taking into account costs on transition from the zero-initial state to the state, which provides these inflows. Therefore, it is necessary to take into account not only the income (results) from the production and transport systems primary activity, but also the corresponding transitional expenses at their projection, not to mention the functioning production and transport systems optimization.

If for the production and transport systems under projection, transitional expenses depend only on the final state (at total – zero – initial state), then for production and transport systems, which are already functioning, transitional expenses depend both on final and on current (initial for optimization procedure) state, which can be different too. In this sense the production and transport system, which is being projected, can be considered as a partial case of the production and transport system, which is functioning, – at the fixed (zero) initial state.

The transitional expenses account reflects also the known lag of the production and transport (as well as any economic) system, and the impossibility of its quick and cheap transfer from one state to another one.

Therefore, the purpose of this article is clarification of influence of initial states and expenses of their transformation into production and transport systems optimal plans.

## RESULTS AND DISCUSSION

The production and transport system activity optimization problem without the account of transitional expenses consists in finding of the extremum of some criterion function  $F$ , which depends on a possible system state  $x$  – at the corresponding restrictions on  $x$ . The production and transport system profit

maximization  $F$  as a function of production volume  $x \geq 0$ , which is produced in it and delivered to consumers, can be a simple and rather widespread example of such statement:

$$F(x) \rightarrow \max_{x \geq 0}. \quad (1)$$

The profit function  $F(x)$  is usually convex up, at first it grows (more and more slowly) with increase in production volume  $x$ , reaches its maximum at the point  $x^*$ , for which the condition  $F'(x) = 0$  is fulfilled, then decreases (Figure 1) due to reduction of the specific income from production additional units' realization and/or growth of specific costs on their production and transportation.

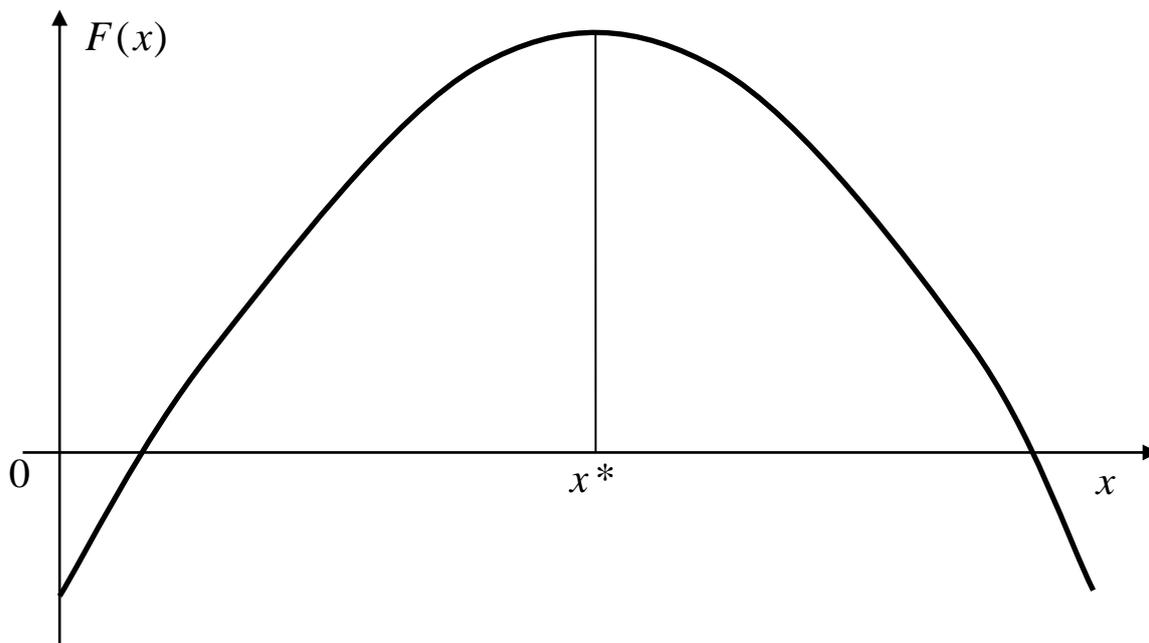


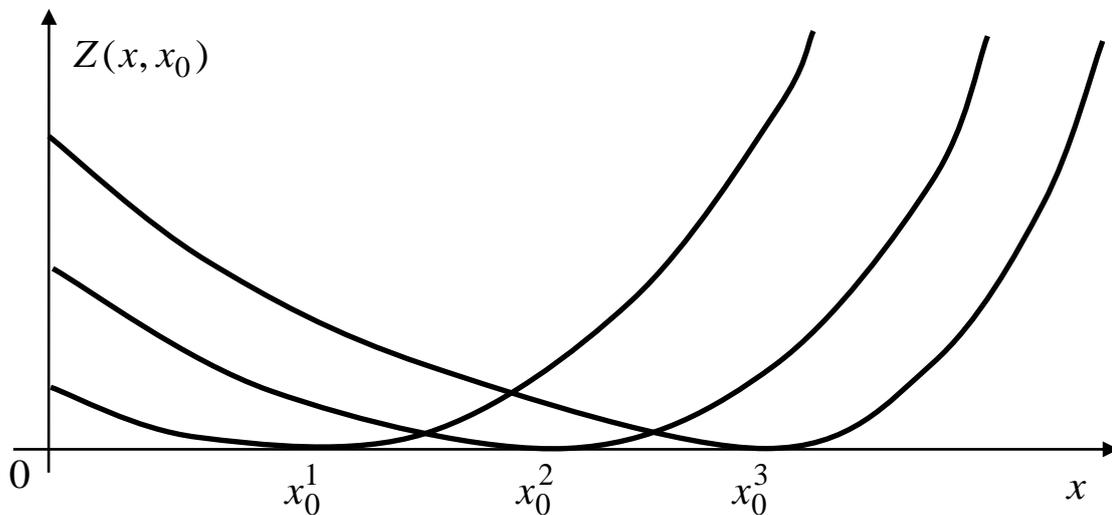
Figure 1. Finding of the production and transport system optimum plan  $x^*$ , excluding transitional expenses

Therefore, at optimization of production transport system excluding transitional expenses it is necessary first to find its optimum production volume  $x^*$ , and then to pass to it from any initial state somehow free of charge.

For accounting of the initial state (reached production and product transportation volume) of production and transport system  $x_0$  and of transitional (to the desirable state) expenses it is necessary to construct the expenses function  $Z(x, x_0)$  on change of production volume from the value  $x_0$  into  $x$ . The basic type of such transitional expenses' dependences at different options of the initial state  $x_0$  is shown in Figure 2.

Note that transitional expenses  $Z(x, x_0)$  grow with removal of the point  $x$  from  $x_0$ , i.e. if  $0 \leq x \leq x_0$ ,  $Z(x, x_0)$  falls down on  $x$ , and if  $x \geq x_0$  – it grows on  $x$ .  $Z(x, x_0)$  grows quicker, than falls down, as the increase in production volume requires more significant expenses, than its reduction (but volume reduction is also connected with certain nonzero expenses). Function  $Z(x, x_0)$  is convex down on  $x$  at given  $x_0$  due to the assumption that each subsequent deviation from  $x_0$  requires higher expenses, than

previous one. With increase of  $x_0$  the growth of  $Z(x, x_0)$  if  $x \geq x_0$  becomes faster, as at large initial production volumes their further increase demands more essential expenses.



**Figure 2.** The basic type of dependences of transitional expenses for change of production volumes of the production and transport system from  $x_0^1, x_0^2, x_0^3$  into  $x$

The identical character sections (descending or growing) of function  $Z(x, x_0)$ , corresponding to different  $x_0$ , don't cross – otherwise transition to a certain state from the different initial states, which are on one side from it  $x_0^1, x_0^2 \leq x$  or  $x_0^1, x_0^2 \geq x$ , could be carried out with identical expenses. Sections of different character are just crossed – if  $x_0^1 < x < x_0^2$ , the equality of transitional (from the different sides) expenses  $Z(x, x_0^1) = Z(x, x_0^2)$  is possible.

Therefore, taking into account expenses  $Z(x, x_0)$  on transition from the initial state  $x_0$  to the desirable state  $x$  the production and transport system optimization problem (1) turns into

$$F(x) - Z(x, x_0) \rightarrow \max_{x \geq 0}. \quad (2)$$

Respectively the optimality conditions turn as well: from  $F'(x) = 0$  into  $F'(x) = Z'(x, x_0)$ .

Absolutely (without taking into account the transitional expenses) optimum plan

$$x^* = \operatorname{Argmax}_{x \geq 0} F(x)$$

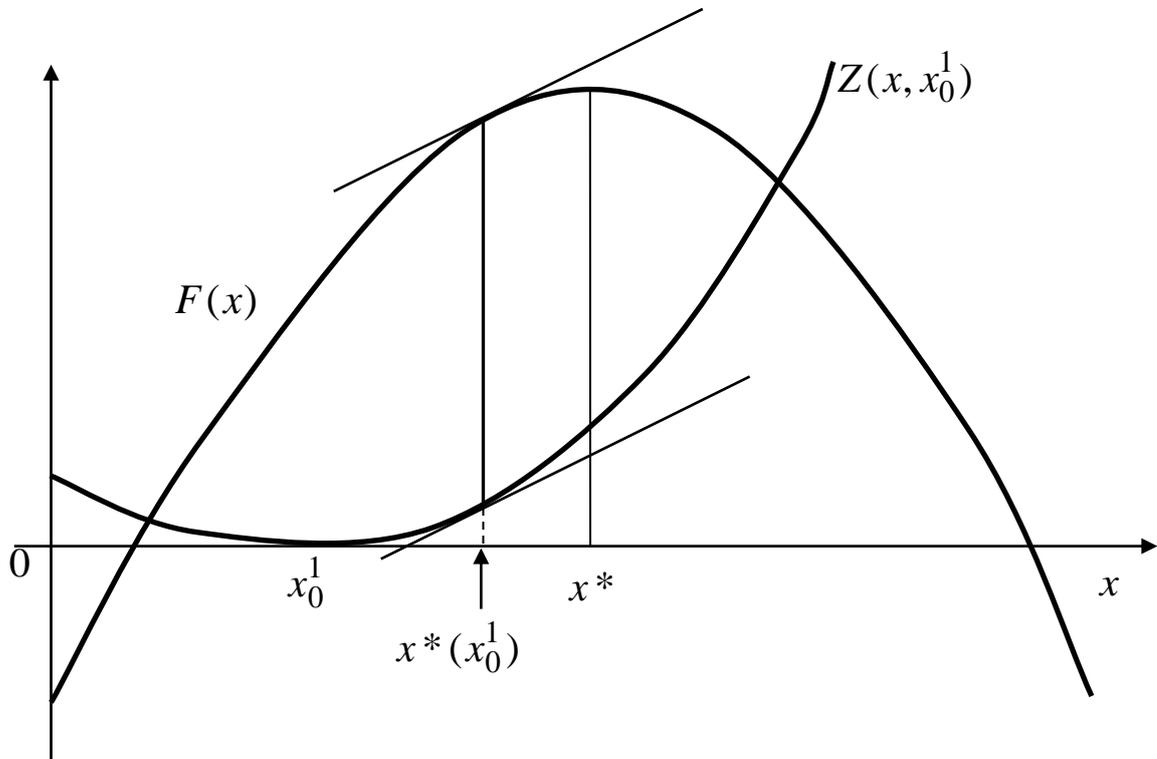
turns into the set of relatively optimum plans

$$x^*(x_0) = \operatorname{Argmax}_{x \geq 0} \{F(x) - Z(x, x_0)\}$$

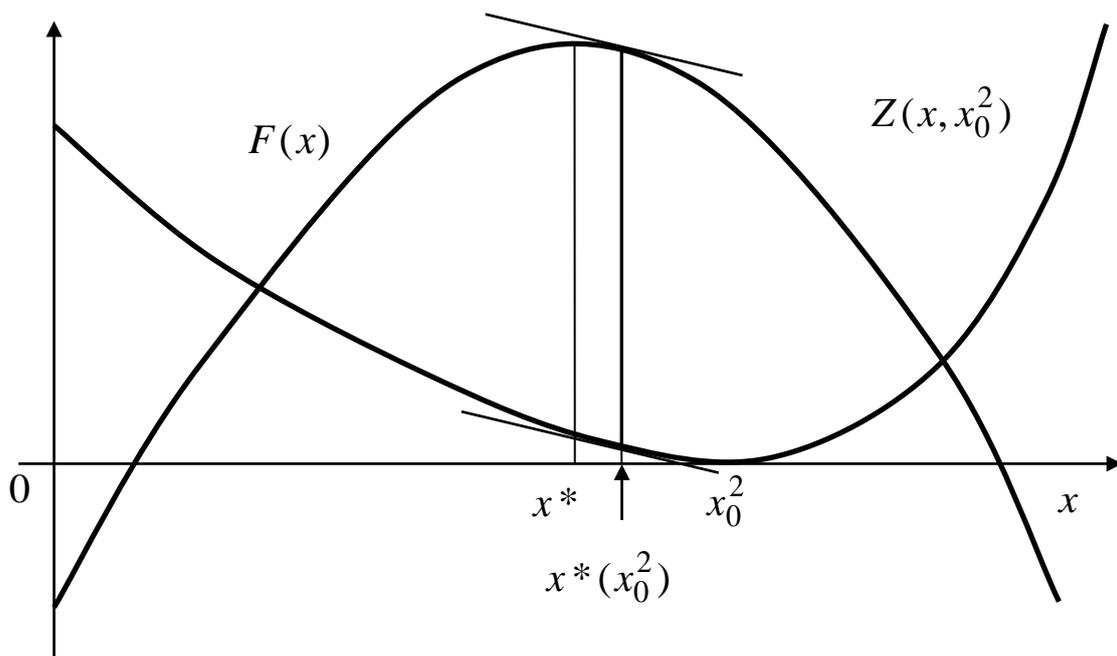
– the certain optimum (concerning this state) plan  $x^*(x_0)$  will correspond to each initial state  $x_0$ .

Note the necessity of discounting of the transitional expenses  $Z(x, x_0)$  with a certain decreasing coefficient for providing the comparison correctness (2), if the profit  $F(x)$  at the state  $x$  of production and transport system can be reached repeatedly (in several periods of time), and expenses on transition to this state are carried out once.

The problem (2) graphic solutions at different initial states  $x_0^1 < x^*$  and  $x_0^2 > x^*$  show (Figures 3, 4), that relatively optimum plans  $x^*(x_0^1)$  and  $x^*(x_0^2)$  (optimum concerning the states  $x_0^1$  and  $x_0^2$ ) differ from absolutely optimum plan  $x^*$ .



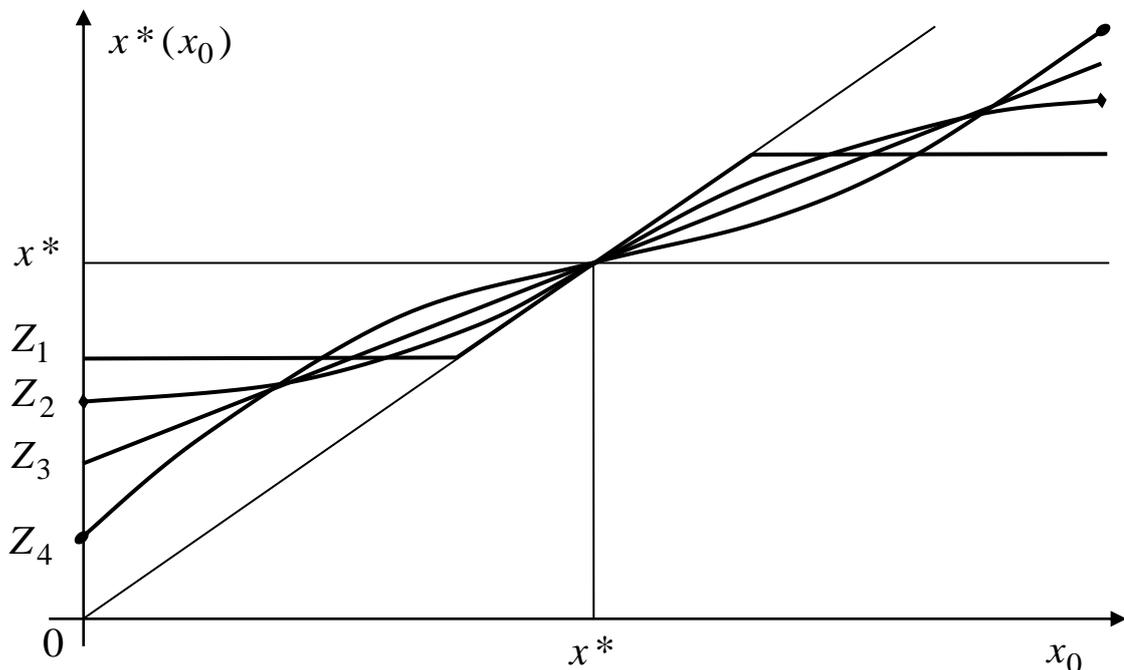
**Figure 3.** Finding of the optimum plan  $x^*(x_0^1)$  of production and transport system with the initial state  $x_0^1 < x^*$ .



**Figure 4.** Finding of the optimum plan  $x^*(x_0^2)$  of production and transport system with the initial state  $x_0^2 > x^*$ .

It is obvious that at the successful initial state  $x_0 = x^*$  the solutions of the problem (1) (absolutely optimum plan) and (2) (relatively optimum plan) will coincide. If  $x_0^1 < x^*$ ,  $x^*(x_0^1) \in [x_0^1, x^*]$ , if  $x_0^2 > x^*$   $x^*(x_0^2) \in [x^*, x_0^2]$ , i.e. the relatively optimum plan is between the initial state and absolutely optimum plan.

The basic type of dependences of relatively optimum plans on the initial state of production and transport system at the transitional expenses different functions is shown in Figure 5. All functions  $x^*(x_0)$  are not decreasing and they are located only in two sectors. Depending on the ratio of profit functions  $F(x)$  characteristics and the transitional expenses  $Z(x, x_0)$  characteristics the functions  $x^*(x_0)$  can be convex up or down (with the inflection point  $x_0 = x^*$ ), linear, partly linear.



**Figure 5.** The basic type of dependences of relatively optimum plans on the production and transport system initial state at the transitional expenses different functions

In limiting cases:

$Z(x, x_0) = 0 \Rightarrow x^*(x_0) = x^*$  (sectors horizontal border) – in the absence of transitional expenses the absolutely optimum plan  $x^*$  is reached from any initial  $x_0$ ;

$Z(x, x_0) = \infty$  (or  $Z'_x(0,0) \geq F'_x(0)$ )  $\Rightarrow x^*(x_0) = x_0$  (sectors inclined border) – at infinitely high transitional expenses every initial state will be relatively optimum (concerning itself), any transitions become inexpedient.

It is easy to receive four basic transitional cases (Figure 5) on the simple numerical example. Let  $F(x) = 8 - (x - 3)^2$ , then  $F'(x) = 6 - 2x = 0$ ,  $x^* = 3$ ,  $F''(x) = -2 < 0$ , in the point  $x^* = 3$  the maximum  $F(x)$  is reached.

We will define on the interval  $x_0 \in [0; 3]$  if  $x \geq x_0$  functions of the transitional expenses.

$$Z_1(x, x_0) = x - x_0, Z'_1 = 1 = F' = 6 - 2x, \text{ from here}$$

$$x_1^*(x_0) = \begin{cases} 2,5, & 0 \leq x_0 \leq 2,5; \\ x_0, & 2,5 \leq x_0 \leq 3. \end{cases}$$

At transitional expenses linear function  $Z_1(x, x_0)$  the dependence of relatively optimum plans  $x_1^*(x_0)$  is partly linear. To the certain limiting border  $0 \leq x_0 \leq 2,5$  all initial states should be transferred in the relatively optimum plan  $x_1^*(x_0) = 2,5$ ; after this limiting border (if  $2,5 \leq x_0 \leq 3$ ) initial states become relatively optimum (the benefit in profit won't compensate the loss on transitional expenses any longer) and don't require any transformations.

$$Z_2(x, x_0) = (x - x_0)^{3/2} = \sqrt{(x - x_0)^3}, Z_2' = 3/2\sqrt{x - x_0} = F' = 6 - 2x, \text{ from here}$$

$$x_2^*(x_0) = (105 - \sqrt{1809 - 576x_0})/32 \geq x_0 \text{ if } 0 \leq x_0 \leq 3, \text{ function } x_2^*(x_0) \text{ is convex down.}$$

$$Z_3(x, x_0) = (x - x_0)^2, Z_3' = 2 \cdot (x - x_0) = F' = 6 - 2x, \text{ from here}$$

$$x_3^*(x_0) = (x_0 + 3)/2 \geq x_0 \text{ if } 0 \leq x_0 \leq 3, \text{ function } x_3^*(x_0) \text{ is linear.}$$

$$Z_4(x, x_0) = (x - x_0)^3, Z_4' = 3 \cdot (x - x_0)^2 = F' = 6 - 2x, \text{ from here}$$

$$x_4^*(x_0) = (3x_0 - 1 + \sqrt{19 - 6x_0})/3 \geq x_0 \text{ if } 0 \leq x_0 \leq 3, \text{ function } x_4^*(x_0) \text{ is convex up.}$$

Coming back from the reviewed numerical example to the general case, we will define function

$$G(x_0) = \max_{x \geq 0} \{F(x) - Z(x, x_0)\} \quad (3)$$

as the maximum production and transport system profit, achievable from the initial point  $x_0$ , taking into account the transitional expenses. The basic type of such functions at different transitional expenses functions  $Z(x, x_0)$  is shown in Figure 6.

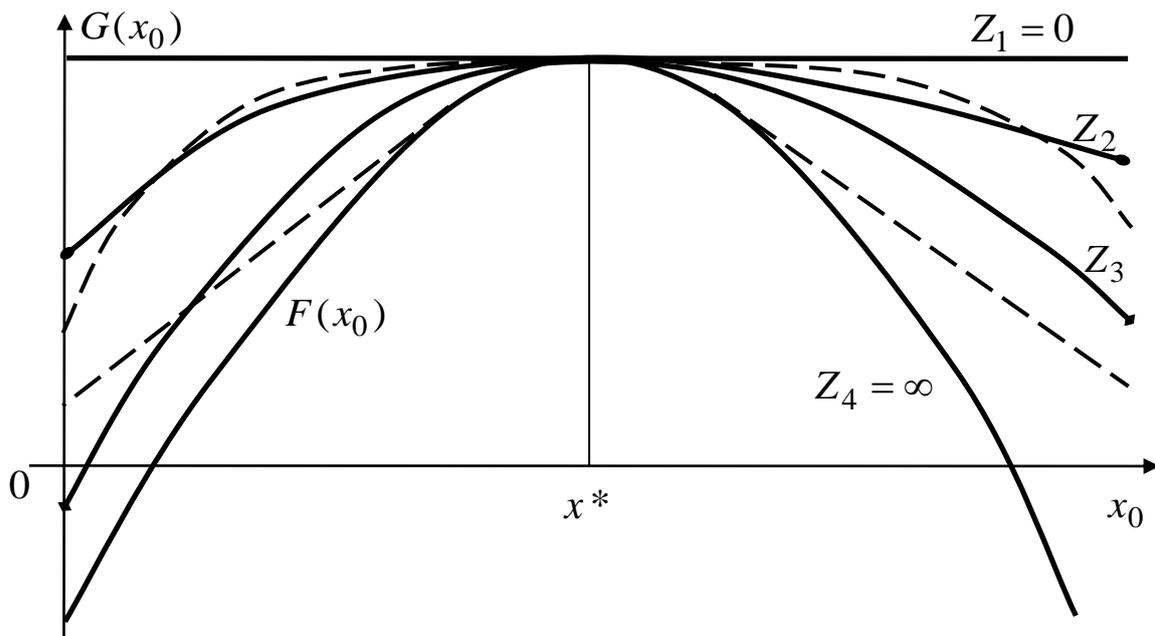


Figure 6. The basic type of dependences of the maximum achievable production and transport system profit on its initial state at different transitional expenses functions



In limiting cases:

in the transitional expenses absence  $Z_1(x, x_0) = 0$   $G_1(x_0) = F(x^*)$ , i.e. from any initial state  $x_0$  the profit  $F(x^*)$  of the absolutely optimum plan  $x^*$  is reached;

at infinitely high transitional expenses  $Z_4(x, x_0) = \infty$   $G_4(x_0) = F(x_0)$  – no changes of any initial state  $x_0$  are inexpedient; you have to be satisfied with profit  $F(x_0)$  of this state.

For intermediate situations  $\forall x \neq x_0$   $0 < Z_2(x, x_0) < Z_3(x, x_0) < \infty$  inequalities  $\forall x_0$   $G_1(x_0) = F(x^*) \geq G_2(x_0) \geq G_3(x_0) \geq G_4(x_0) = F(x_0)$  are carried out. Increase of profit  $G_2(x_0) - F(x_0)$  or  $G_3(x_0) - F(x_0)$  is reached due to the transition possibility at finding  $G_2(x_0)$  or  $G_3(x_0)$  from the initial state  $x_0$  to corresponding relatively optimum state  $x_2^*(x_0)$  or  $x_3^*(x_0)$ , with the surplus compensation for expenses on this transition. Therefore functions  $G_2(x_0)$ ,  $G_3(x_0)$  become more flat in comparison with  $F(x_0)$ , moreover their right (from  $x^*$ ) part even more flat, than left one (Figure 6) as transitional expenses  $Z(x, x_0)$  at production volumes reduction are lower, than at their increase (Figure 2). Only if  $x_0 = x^*$  the values of all functions  $G_1(x^*) = G_2(x^*) = G_3(x^*) = G_4(x^*) = F(x^*)$  coincide.

Note that unlike functions  $x^*(x_0)$ , which could be convex both up and down (Figure 5), functions  $G(x_0)$  are convex up (Figure 6), but characteristics of this convexity up can be different (the options are shown by dotted lines in Figure 6) – depending on the transitional expenses' functions  $Z(x, x_0)$  peculiarities. In particular, the function  $G(x_0)$  linearity sectors, corresponding to horizontal sectors of partly-linear function  $x^*(x_0)$  in Figure 5, are possible if transitional expenses function  $Z(x, x_0)$  is linear.

Now we will apply the stated general ideas to the classical transport problem

$$f(x) = \sum_{i=1}^m \sum_{j=1}^n c_{ij} \cdot x_{ij} \rightarrow \min_{\{x_{ij}\}} \quad (4)$$

$$\sum_{j=1}^n x_{ij} = a_i, i = 1, \dots, m \quad (5)$$

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, \dots, n \quad (6)$$

$$x_{ij} \geq 0, i = 1, \dots, m, j = 1, \dots, n, \quad (7)$$

where  $m$  – the number of producers in production and transport system,

$n$  – the number of consumers,

$a_i$  – the production volume of the producer  $i$ ,  $i = 1, \dots, m$ ,

$b_j$  – the production necessity for the consumer  $j$ ,  $j = 1, \dots, n$ ,

$c_{ij}$  – the specific costs on production transportation from the producer  $i$  to the consumer  $j$ ,

$i = 1, \dots, m, j = 1, \dots, n$ ,



$x = \{x_{ij}\}, i = 1, \dots, m, j = 1, \dots, n$  – the production and transport system state,

$x_{ij}$  – the product transportation volume from the producer  $i$  to the consumer  $j, i = 1, \dots, m, j = 1, \dots, n$ .

Let  $x_0 = \{x_{ij}^0\}$  be the initial state of such elementary production and transport system, then criterion function (4) taking into account expenses not only on transportation  $f(x)$ , but also on transition from the initial state  $x_0$  to the actual one  $x$  in the amount  $Z(x, x_0)$  will turn into

$$f(x) + Z(x, x_0) \rightarrow \min_{x \in (5)-(7)} \quad (8)$$

The transitional expenses function  $Z(x, x_0)$  can be determined in different ways. From the point of view of economic sense, the transitional expenses are connected, mainly, with the transportation's organization on the new, earlier not involved destinations, and therefore it is possible to introduce the corresponding indicative variables

$$\delta_{ij}(x_{ij}, x_{ij}^0) = \begin{cases} 1, & \text{if } x_{ij} > 0, x_{ij}^0 = 0 \\ 0 & \text{if not} \end{cases}$$

and expenses  $k_{ij}$  on the transportation's organization from the producer  $i$  to the consumer  $j, i = 1, \dots, m, j = 1, \dots, n$ . Then

$$Z(x, x_0) = \sum_{i=1}^m \sum_{j=1}^n \delta_{ij}(x_{ij}, x_{ij}^0) \cdot k_{ij}. \quad (9)$$

At such transitional expenses formation, the volume is not of the crucial importance, but the transportation fact is, and in the relatively optimum plan we should expect the minimum quantity of new transportations (busy cells) at their maximum possible volumes (since we transport somewhere – let's do it at full scale). The criterion function (8) discontinuity due to the discontinuous component (9) is the drawback of this approach realization, which complicates the corresponding computing procedures.

From the mathematical point of view, it is expedient to determine the distance between the initial state  $x_0$  and actual one  $x$  as distance between two points

$$d(x, x_0) = \sqrt{\sum_{i=1}^m \sum_{j=1}^n (x_{ij} - x_{ij}^0)^2} \quad (10)$$

then

$$Z(x, x_0) = k \cdot d(x, x_0), \quad (11)$$

where  $k$  – average specific transitional expenses.

Considering in (10) the expression  $s_{ij}(x_{ij}, x_{ij}^0) = \max\{x_{ij} - x_{ij}^0; 0\}$  instead of  $x_{ij} - x_{ij}^0$ , it is possible to take into account only the ways of transportation volumes growth in transitional expenses (11) formation. Unlike the approach (9), here transitional expenses (11) and total expenses (8) become continuous; however, the particularities of function of distance between states (10) will lead to a large number of insignificant

transportations (the filled cells) that contradicts the problem economic sense at interpretation of the transitional expenses as expenses on the new transportation organization.

The transitional expenses determination as

$$Z(x, x_0) = \sum_{i=1}^m \sum_{j=1}^n s_{ij}(x_{ij}, x_{ij}^0) \cdot k_{ij}. \quad (12)$$

can be a certain compromise between economic (9) and mathematical (10) - (11) approaches.

In method (12) the transitional and total expenses continuity remains; the transitional expenses don't depend on the new transportations fact, as in (9), but on their volume; the difference in transportation volumes  $s_{ij}(x_{ij}, x_{ij}^0)$  has more transparent economic sense, than mathematical distance between states (10).

For simplicity instead of transportation organization specific expenses  $\{k_{ij}\}$ , differentiated in the transportation destinations it is possible, as in (11), to consider the uniform average value  $k$ . We will define the corresponding dependences taking into account these transitional expenses characteristic  $k$ :

transitional expenses

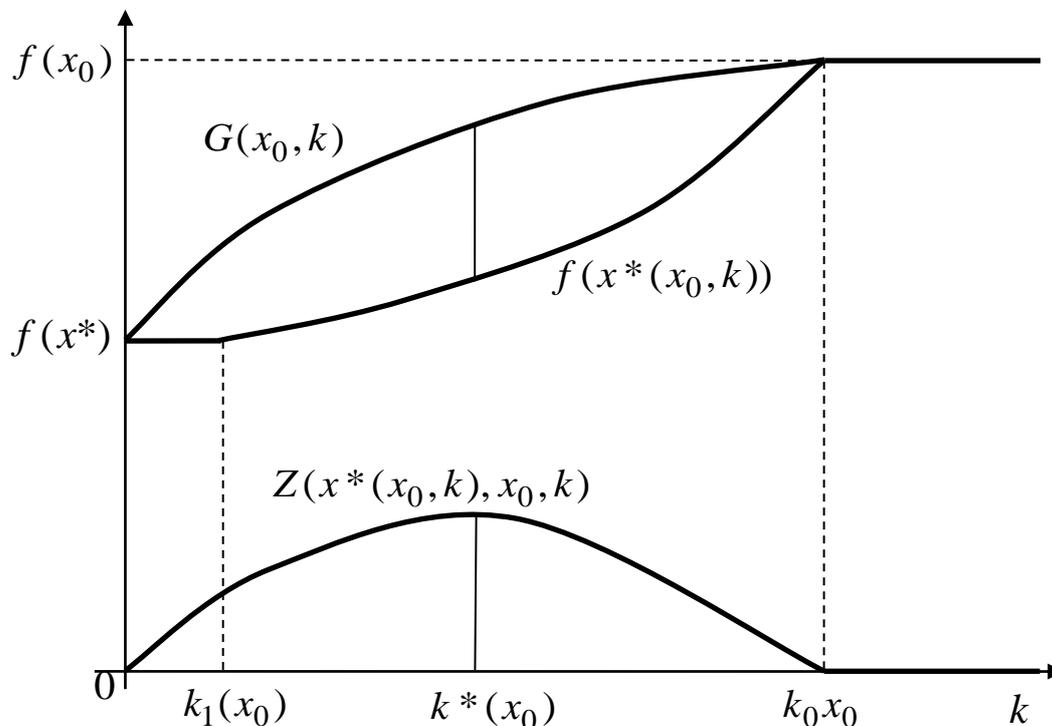
$$Z(x, x_0, k) = k \cdot \sum_{i=1}^m \sum_{j=1}^n s_{ij}(x_{ij}, x_{ij}^0), \quad (13)$$

relatively optimum state

$$x^*(x_0, k) = \text{Arg} \min_{x \in (5)-(7)} \{f(x) + Z(x, x_0, k)\}, \quad (14)$$

the total expenses value (on transportations and on transition to relatively optimum state), minimum achievable from the initial state.

$$G(x_0, k) = \min_{x \in (5)-(7)} \{f(x) + Z(x, x_0, k)\} = f(x^*(x_0, k)) + Z(x^*(x_0, k), x_0, k). \quad (15)$$



**Figure 7.** The basic type of dependences of total expenses  $G$ , expenses on transportation  $f$  and on transition between states  $Z$  on the specific transitional expenses  $k$



The basic type of dependences of this value and its components on the specific transitional expenses  $k$  is shown in Figure 7.

At zero specific transitional expenses  $k = 0$ , it is possible to pass from any initial state  $x_0$  to absolutely optimum state  $x^*$  free of charge. At low specific transitional expenses  $k \leq k_1(x_0)$  the transition is carried out all the same to the absolutely optimum state  $x^*$  (expenses on transportation remain at the same level  $f(x^*)$ ), however already with certain transitional expenses  $Z(x^*, x_0, k)$  due to which – if transportation expenses are constant – total expenses  $G(x_0, k)$  also grow.

At higher  $k_1(x_0)$  specific transitional expenses  $k$ , the transition will be already carried out not to absolutely, but to relative optimum state  $x^*(x_0, k)$ , closer to the initial one, with higher expenses namely on transportation  $f(x^*(x_0, k))$ . At the same time the transitional expenses  $Z(x^*(x_0, k), x_0, k)$  grow (but more and more slowly) to  $k = k^*(x_0)$  (the transition itself from  $x_0$  to  $x^*(x_0, k)$  becomes shorter; however, the specific expenses on it grow rather quickly). Therefore, total expenses  $G(x_0, k)$  also grow quickly enough on this interval.

If  $k = k^*(x_0)$  the transitional expenses  $Z(x^*(x_0, k), x_0, k)$  reach their maximum value (which is equal to the maximum distance between curves of transportation costs  $f(x^*(x_0, k))$  and total expenses  $G(x_0, k)$ ), then they begin to fall down (as approach rates of relatively optimum state  $x^*(x_0, k)$  to the initial one  $x_0$  already advance growth rates of the specific transitional expenses  $k$ ). But expenses on transportation  $f(x^*(x_0, k))$  due to fast approach of relatively optimum state  $x^*(x_0, k)$  to initial one  $x_0$  grow considerably faster, than transitional expenses  $Z(x^*(x_0, k), x_0, k)$  fall down, and therefore total expenses  $G(x_0, k)$  though slowly, but grow too.

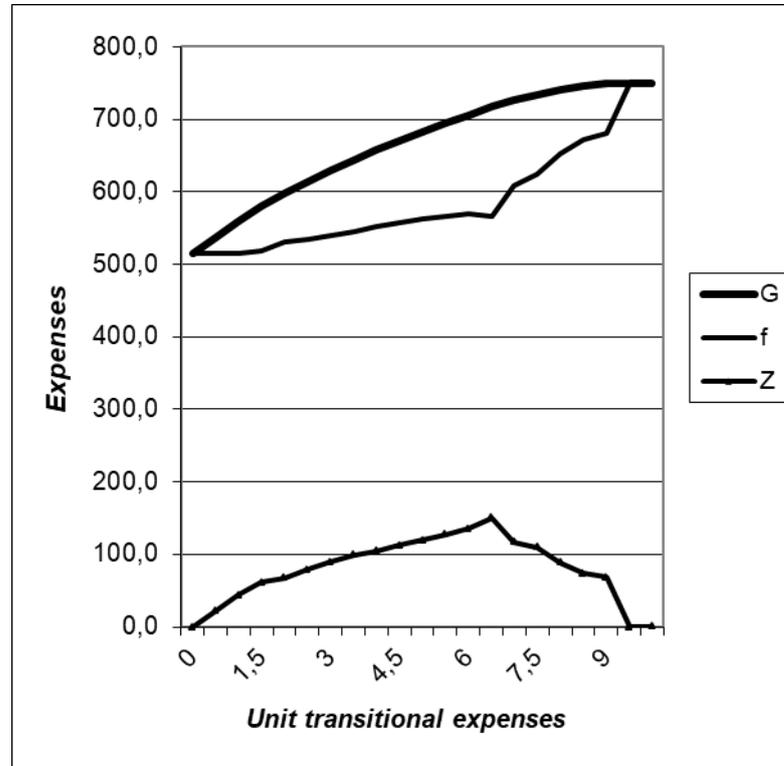
Finally, if the specific transitional expenses  $k \geq k_0(x_0)$  are very high, all possible transitions from the initial state  $x_0$  are economically inexpedient, it becomes relatively optimum (concerning itself), therefore transitional expenses turn into zero, and total expenses  $G(x_0, k)$  are equal to expenses on transportation  $f(x_0)$  in the initial state.

Pay attention that the main values of the specific transitional expenses  $k_1(x_0)$ ,  $k^*(x_0)$ ,  $k_0(x_0)$  depend on the initial state  $x_0$ .

Note also the basic (smoothed) type of the corresponding dependences is shown in Figure 7. While carrying out the computing experiments series by means of the option "Solver" of the MS Excel package on transport problems numerical examples the established general tendencies were observed rather accurately (especially concerning total expenses  $G(x_0, k)$ ), however concerning components – i.e. costs on transportation and the transitional expenses – the certain not smooth effects (Figure 8), caused by specifics of linear programming problems conditions polyhedrons, took place.

Note the connection of the considered range of problems with monitoring, popular at the present stage, – tracking of systems functioning processes and accepted plans implementation. Therefore, at deviations identification it isn't necessary to provide return to the earlier calculated optimum plan mechanically

(irrespective of the transitional expenses, i.e. at any cost), but to correct this optimum plan taking into account the transitional expenses, receiving the new desirable (relatively optimum) plan.



**Figure 8.** The empirical dependences example of total expenses  $G$ , expenses on transportation  $f$  and on transition between states  $Z$  on the specific transitional expenses  $k$

We will draw another parallel between the considered transitional expenses (on system transfer in the desirable state) and transportation costs on delivery (if  $x_0 < x^*(x_0)$ ) or production pickup (if  $x_0 > x^*(x_0)$ ). In economic literature both transfer costs, and transportation costs are often neglected for their allegedly insignificance, however actually they influence optimum plans significantly. It may be said as well that such expenses act as the certain smoothing factor, shifting relatively optimum state (which takes them into account) from absolutely optimum (at their absence) towards initial one.

The dependences of production and transport system income  $D(x)$  and total expenses  $Z(x)$ , including production expenses  $Z_1(x)$  and transport expenses  $Z_2(x)$  on the production volume  $x$  are shown in Figure 9.

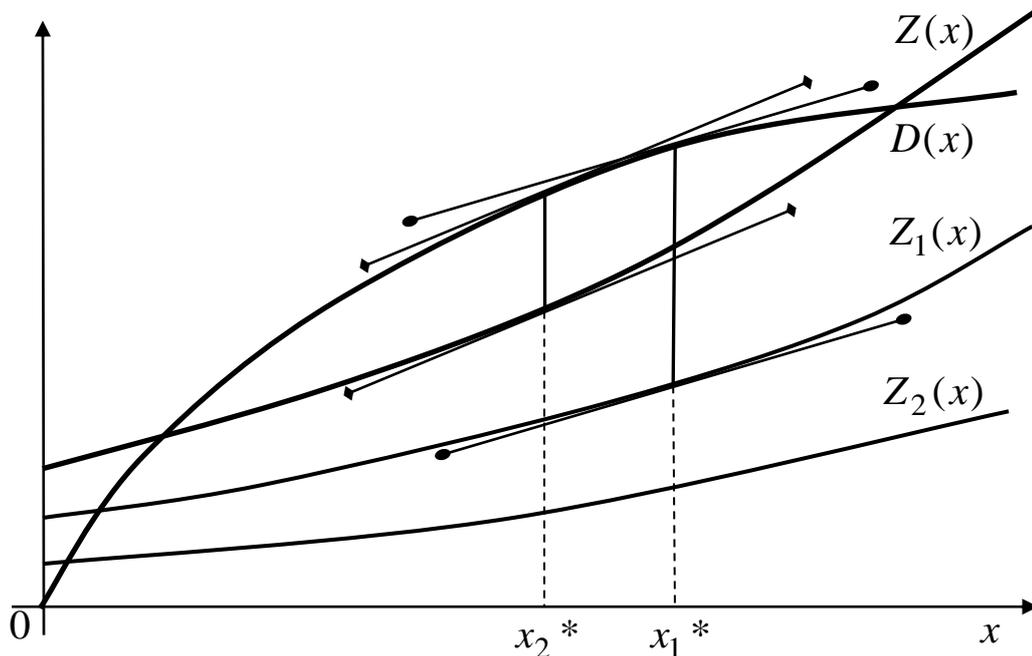
Without account (or at absence) of transportation costs, the production and transport system profit will be formed as  $F_1(x) = D(x) - Z_1(x)$ , its maximum will be reached in the point  $x_1^*$ , in which  $D'(x_1^*) = Z_1'(x_1^*)$  (in Figure 9 tangents to curves  $D(x)$  and  $Z_1(x)$  in the point  $x_1^*$  are parallel).

Taking into account transportation costs, the profit can be found as  $F(x) = D(x) - Z(x) = D(x) - (Z_1(x) + Z_2(x)) = D(x) - Z_1(x) - Z_2(x)$ , its maximum is reached in the

point  $x_2^* < x_1^*$ , in which  $D'(x_2^*) = Z'(x_2^*)$  (in Figure 9 in the point  $x_2^*$  tangents to curves  $D(x)$  and  $Z(x)$  are parallel).

It is clear, that  $F(x_2^*) = D(x_2^*) - Z_1(x_2^*) - Z_2(x_2^*) < F_1(x_1^*) = D(x_1^*) - Z_1(x_1^*)$ , i.e. in the point  $x_2^*$ , optimal at the transportation costs accounting, the production and transport system profit is reduced in comparison with the point  $x_1^*$ , which is ideally optimal (if there really weren't transportation costs).

However,  $F(x_2^*) = D(x_2^*) - Z_1(x_2^*) - Z_2(x_2^*) > F(x_1^*) = D(x_1^*) - Z_1(x_1^*) - Z_2(x_1^*)$ , i.e. in the real case (taking into account transportation costs) the profit in the point  $x_2^*$  is higher, than in calculated for point  $x_1^*$ , which is the ideal case.



**Figure 9.** Production and transport system optimization at absence of transportation costs and at taking them into account

## CONCLUSIONS

Therefore, it is better to correct the optimum plans taking into account transportation costs (than to insist on the optimums, received without their account) – then the real income will decrease in comparison with ideal one, but not so much as it would decrease in the transportation costs presence at orientation to the ideal optimum.

In the same way the transitional expenses accounting reduces the profit in comparison with a hypothetical case of their absence, however at their actual existence the profit in the respectively corrected relatively optimum state will be higher, than in absolutely optimum.

Thus, introduction of initial states and transitional expenses to the models of production and transport systems optimization will promote the increase of their adequacy, flexibility and adaptability that is especially actual in the economy market transformation conditions.



From the mathematical point of view, the accounting of initial states and the transitional expenses, on the one hand, complicates the corresponding models and computing processes, however on the other hand – does the models even more interesting, creates additional uncommon effects.

The further consideration of not only transitional expenses, but also of time for transition from the initial state to actual ones is supposed as namely the time factor is of crucial importance for the transport component of the production and transport systems (Voynarenko & Kholodenko, 2019).

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All authors have read and agreed to the published version of the manuscript.

### **Informed Consent Statement:**

Informed consent was obtained from all the participants involved in the study.

**Data Availability Statement:** The data presented in this study are available on request from the corresponding author. The data are not publicly available due to privacy issues.

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### **Conflict of interests**

The authors declare no conflict of interests.

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