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MOMENT EQUATIONS FOR STOCHASTIC SYSTEM SPECIAL KIND AS INSTRUMENT IN APPLY PROBLEM

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ABSTRACT

In paper considered the method of constructing moment equations for random solution of systems of nonlinear differential and difference equations, the right part of which depends on the stochastic process. Torque equations are constructed in the presence of jumps in solutions. For a system of differential equations with random coefficients, the case when the heterogeneous part of the system contains random processes such as white noise is considered.
The ideas of A.M. Kolmogorov and V.I. Zubov on the analytical definition of random processes have been developed. In particular, non-Markov processes are investigated, which are determined by systems of linear differential equations with a delay in the argument.
With the help of stochastic operators, fundamentally new results were obtained for non-Markov random processes, from which the main known results for Markov processes emerge. Methods and algorithms of analytical determination of finite-valued and infinite-digit random processes are proposed.
The methods of studying the behaviours of the matrix of the second moments of some important classes of stochastic systems of equations are given because many optimization problems are reduced to the minimization of such a matrix. The substantiation of difference approximation for solving some types of differential equations used for the numerical solution of problems is carried out.

Keywords: *moments first and second order, white noise, differential equations, numerical solution*

JEL classification: C60, C40, H30

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INTRODUCTION

In modern applied studies, probabilities models are widely used. Among such models, classes of systems of ordinary differential and difference equations are practically important, the coefficients of which are random processes. Therefore, the study of probability models led to the creation of various current directions in the theory of stability and the closely related theory of optimal control of random processes.

The theoretical foundations of sustainability research for systems of differential equations with random processes were initiated by A.M. Kolmogorov. In the future, the approaches of A.M. Kolmogorov developed in their works D. Bertrand, Ghichmann, V.I. Zubov, M.M. Krasovsky, V.S. Korolyuk, I.M. Stratonovich, Ito, A.M. Tikhonov, D.G. Korenovsky, V.B. Kolmanovsky, R.Z. Hasminsky, A.V. Skorokhod, I.E. V.Y. Valeev, G.M. Milstein and others. The works of most researchers are based either on the study of the Fokker–Planck–Kolmogorov type equation and the probabilistic properties of solving stochastic differential equations, or on the analysis of momentary equations followed by the application of O.M. Lyapunov's methods.

The developed theoretical, methodological and applied recommendations of this dissertation make it possible to expand the modern mathematical apparatus for optimizing dynamical systems in conditions of uncertainty and conflict situations. In particular, a number of problems related to political technologies, and banking are solved.

METHODOLOGY

The paper uses a mathematical apparatus of the method of momentary equations, methods of functions and functionals of Lyapunov, general theory of random processes, theory of differential, difference and integral equations.

RESULTS

On a probabilise basis $(\Omega, \Im, P, F \equiv \{F_t, t \geq 0\})$ consider the system of stochastic differential equations

$$dX(t) = AX(t)dt + Bdw(t) \qquad (1)$$

With initial conditions $X(0) = \phi(\omega)$, де $w(t) = w(t,\omega): [0,\tau] \times \Omega \to E$ — Winer random process, A,B — some matrices *on* dimension $m \times m$, a $\phi(\omega)$ — random variable with its distribution law $F_\phi(x)$.

Theorem 1. *The matrix of second order $D(t) = \langle X(t)X^*(t)\rangle$ of strong solution $X(t) \equiv X(t,\omega)$ system of equations (1) satisfies the equation*

$$\frac{dD(t)}{dt} = AD(t) + D(t)A^* + BB^*. \qquad (2.)$$

The solution of the system (2) is based on the formula

$$D(t) = e^{At}D(0)e^{A^*t} + \int_0^t e^{A(t-s)}\, BB^{*e^{A^*(t-s)}} ds. \qquad (3)$$

If $\lim_{t\to+\infty} e^{At} = 0$, *then we obtain the value of matrix* $D = \lim_{t\to+\infty} D(t)$ *from formula*

$$D = \int_0^\infty e^{At}\, BB^* e^{A^*t} dt.$$

Proof. *1 algorithm.* The auxiliary system is considering

$$\frac{dX(t)}{dt} = AX(t) + B\xi_\lambda(t),$$

here $\xi_\lambda(t)$ — Markov process, with value $\pm\sqrt{\lambda}$ and probabilities

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$$p_1(t) = P\{\omega: \xi(t) = \sqrt{\lambda}\}, p_2(t) = P\{\omega: \xi(t) = -\sqrt{\lambda}\},$$

Which satisfied the system equations?

$$\frac{dp_1(t)}{dt} = -\lambda p_1(t) + \lambda p_2(t), \frac{dp_2(t)}{dt} = \lambda p_1(t) - \lambda p_2(t).$$

Now constructing the system of moment equations

$$\frac{dM_1(t)}{dt} = AM_1(t) + B \cdot 0{,}5\sqrt{\lambda} - \lambda M_1(t) + \lambda M_2(t),$$

$$\frac{dM_2(t)}{dt} = AM_2(t) - B \cdot 0{,}5\sqrt{\lambda} + \lambda M_1(t) - \lambda M_2(t), M_k(0) = 0 \qquad k = 1{,}2.,$$

From this system we find

$$\frac{d(M_1(t)+M_2(t)}{dt} = A(M_1(t) + M_2(t)), M_1(t) + M_2(t) \equiv 0.$$

For particular moments we have next system

$$\frac{dD_1(t)}{dt} = AD_1(t) + D_1(t)A^* - \lambda D_1(t) + \lambda D_2(t) + AM_1(t)\sqrt{\lambda}B^* + B\sqrt{\lambda}M_1^*(t)A^* + 0{,}5BB^*,$$

$$\frac{dD_2(t)}{dt} = AD_2(t) + D_2(t)A^* + \lambda D_2(t) - \lambda D_2(t) + AM_2(t)\sqrt{\lambda}B^* + B\sqrt{\lambda}M_2^*(t)A^* + 0{,}5BB^*, D_k(0) = 0.$$

$$D(t) = \langle X(t)X^*(t)\rangle = D_1(t) + D_2(t)$$

We obtain matrix equation (2)

2 algorithm. From formula Ito [6]

$$X(t+dt) = X(t) + dX(t) + \frac{1}{2}d^2X(t) + \ldots = X(t) + \big(AX(t)dt + Bdw(t)\big) +$$

$$+\frac{1}{2}\big(A(AX)(t)dt + Bdw(t)\big)dt + \ldots = X(t) + AX(t)dt + Bdw(t) + O\left((dt)^{3/2}\right).$$

$$X(t+dt) = X^*(t+dt) = X(t)X^*(t) + AX(t)X^*(t)dt$$

$$+X(t)X^*(t)A^*dt + X(t)B^*dw(t) = BX^*(t)dw(t) + BB^*\big(dw(t1)\big)^2 +$$

$$+O(dt)^{3/2} = AD(t)dt + D(t)A^*dt + BB^*dt + O(dt).$$

If $dt \to 0$ obtain (2).

Rewrite system (1) in form

$$\frac{dX(t)}{dt} = AX(t) + Bv(t),$$

$v(t)$ — white noise. If $Re\,\lambda_j\,(A) < 0$ we find solution

$$X(t) = \int_0^\infty e^{A(t-\tau)}\, Bv(\tau)d\tau.$$

And for matrix second order we have:

$$\boldsymbol{D(t) = \langle X(t)X^*(t)\rangle = \left\langle \int_0^t e^{A(t-\tau)}\, Bv(\tau)d\tau \times \int_0^t B^* e^{A^*(t-s)}\, v(s)ds \right\rangle}$$

$$\boldsymbol{= \int_0^t e^{A(t-\tau)}\, BB^* d\tau \int_0^t e^{A^*(t-s)}\, \delta(\tau - s)ds =}$$

$$= \int_0^t \boldsymbol{e^{A(t-\tau)} BB^* e^{A^*(t-\tau)} d\tau} = \int_0^t \boldsymbol{e^{A\tau} BB^* e^{A^*\tau} d\tau}.$$

Then true (3). If $\lim_{t\to+\infty} e^{At} = 0$, then from equations (2) we have the value of matrix $D(t)$ at $t \to +\infty$.

Theorem 2. *Для системи нелінійних диференціальних стохастичних рівнянь*

$$\frac{dX(t)}{dt} = \Psi\big(t, X(t)\big) + \Phi\big(t, X(t)\big)\nu(t), \tag{4}$$

where $\nu(t)$ *— white noise the system of moment equations has kind*

$$\frac{dM(t)}{dt} = \int_{E_m} \Psi(t,y) f(t,y,h) dy,$$

$$\frac{dD(t)}{dt} = \int_{E_m} \big(\Psi(t,y)y^* + y\Psi^*(t,y) + \Phi(t,y)\Phi^*(t,y)\big) f(t,y,h) dy. \tag{5}$$

Proof. System (4) we rewrite in kind [1]

$$dX(t) = \Psi\big(t, X(t)\big)dt + \Phi\big(t, X(t)\big)dw(t),$$

At first we prove that the system

$$X_{k+1} = X_k + h\Psi(t_k, X_k) + \xi_k\sqrt{h}\Phi(t_k, X_k), t_k = kh, \quad (k = 0,1,2,\dots) \tag{8}$$

Defined difference approximation for the system equating (4), де ξ_k — have value ± 1 *with probability* $p_1 = p_2 = \frac{1}{2}$:

$$x = T\big(t, y, \sqrt{h}\big), T\big(t, y, \sqrt{h}\big) \equiv y + \sqrt{h}\Phi(t,y) + h\Psi(t,y) \tag{9}$$

If $h > 0$ expression (7) has inverse transformation

$$y = S\big(t, x, \sqrt{h}\big), \tag{10}$$

$$S\big(t, x, \sqrt{h}\big) = x - \sqrt{h}\Phi(t,x) - h\Psi(t,x) + h\frac{D\Phi(t,x)}{Dx}\Phi(t,x) + O\left(h^{3/2}\right).$$

If $h = 0$ Jacobian of transformation (9), (10) not equals 0 at all Y i X, therefore

$$det\frac{DT\big(t, y, \sqrt{h}\big)}{Dy} \neq 0, det\frac{DS\big(t, x, \sqrt{h}\big)}{Dx} \neq 0$$

$$\lim_{\|y\|\to\infty} \left\|T\big(t, y, \sqrt{h}\big)\right\| = \infty, \lim_{\|x\|\to\infty} \left\|S\big(t, x, \sqrt{h}\big)\right\| = \infty.$$

Let $f(t\ \ , x, h)$ - density of X_k. The system equations (4) defined the solutions $f(t, x, h)$

$$f(t + h, x, h) = \frac{1}{2} f\Big(t, S\big(t, x, \sqrt{h}\big), h\Big) \cdot \left|det\frac{dS(t,x,\sqrt{h})}{dx}\right| +$$

$$+\frac{1}{2} f\Big(t, S\big(t, x, -\sqrt{h}\big), h\Big) \left|det\frac{dS(t,x,-\sqrt{h})}{dx}\right|.$$

$$f\left(t, x - \sqrt{h}\Phi(t,X) - h\Psi(t,x) + h\frac{d\Phi(t,x)}{dx}\Phi(t,x) + O\left(h^{3/2}\right), h\right) =$$

$$= f(t,x,h) + \frac{df(t,x,h)}{dx}\Big(-\sqrt{h}\Phi(t,x) - h\Psi(t,x) +$$

$$+\frac{d\Phi(t,x)}{dx}\Phi(t,x)\Big) + \frac{1}{2}h\Phi^*(t,x)\frac{d}{dx}\left(\frac{df(t,x,h)}{dx}\right)^* \Phi(t,x) + O\left(h^{3/2}\right).$$

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And

$$det\frac{DS(t,x,\sqrt{h})}{Dx}=det\left(E-\sqrt{h}\frac{D\Phi(t,x)}{Dx}-h\frac{D\Psi(t,x)}{Dx}+\right.$$

$$\left.+h\frac{D}{Dx}\left(\frac{D\Phi(t,x)}{Dx}\Phi(t,x)\right)+O\left(h^{3/2}\right)=det\left(E-\sqrt{h}\frac{D\Phi(t,x)}{Dx}\right)\times\right.$$

$$\times\, det\left(E-h\frac{D\Psi(t,x}{Dx}\right)det\left(E+h\frac{D}{Dx}\left(\frac{D\Phi(t,x)}{Dx}\Phi(t,x)\right)+O\left(h^{3/2}\right)\right).$$

Calculating determinants value

$$det\left(E-h\frac{D\Psi(t,x)}{Dx}=1-hSp\frac{D\Psi(t,x)}{Dx}+O(h^2)=\right.$$

$$=1-hdiv\Psi(t,x)+O(h^2);$$

$$det\left(E+h\frac{D}{Dx}\left(\frac{D\Phi(t,x)}{Dx}\Phi(t,x)\right)\right)=1+hSp\frac{D}{Dx}\left(\frac{D\Phi(t,x)}{Dx}\Phi(t,x)\right)+O(h^2).$$

If

$$det\left(E-\sqrt{h}\frac{D\Phi(t,x)}{Dx}\right)=1-\alpha\sqrt{h}+\beta h+O\left(h^{3/2}\right).$$

then

$$det\left(E+\sqrt{h}\frac{D\Phi(t,x)}{Dx}\right)=1+\alpha\sqrt{h}+h\beta+O\left(h^{3/2}\right),$$

where

$$\alpha=Sp\frac{D\Phi(t,x)}{Dx},$$

$$\beta=\frac{1}{2}\left(Sp\frac{D\Phi(t,x)}{Dx}\right)^2-\frac{1}{2}Sp\left(\frac{D\Phi(t,x)}{Dx}\right)^2.$$

$$det\left(E-\sqrt{h}\frac{D\Phi(t,x)}{Dx}\right)\left(E+\sqrt{h}\frac{D\Phi(t,x)}{Dx}\right)=1+h(2\beta-\alpha^2)+O(h^2).$$

$$det\left(E-\sqrt{h}\frac{D\Phi(t,x)}{Dx}\cdot\frac{D\Phi(t,x)}{Dx}\right)=1-hSp\left(\frac{D\Phi(t,x)}{Dx}\cdot\frac{D\Phi(t,x)}{Dx}\right)+O(h^2)=$$

$$=1+h(2\beta-\alpha^2)+O(h^2)..$$

and

$$det\frac{DS(t,x,\sqrt{h})}{Dx}=1-\sqrt{h}div\Phi(t,x)-hdiv\Psi(t,x)+$$

$$+\frac{h}{2}\left(div\Phi(t,x)\right)^2+\frac{h}{2}\sum_{k,s=1}^{m}\left(\frac{\partial\phi_k(t,x)}{Dx_s}\frac{\partial\phi_s(t,x)}{\partial x_k}+\right.$$

$$\left.+\frac{\partial\phi_s(t,x)}{\partial x_k\partial x_s}\phi_k(t,x)\right)+O\left(h^{3/2}\right).$$

where $\phi_k(t,x)(k=1,\dots,m)$—coordinate of vector $\Phi(t,x)$.

$$f(t,x)=-\lim_{h\to 0}f(t,x,h).$$

This equation coincides with FPK

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$$\frac{\partial f(t,x)}{\partial t} = -\sum_{k=1}^{m} \frac{\partial f(t,x)\phi_k(t,x)}{\partial x_k} + \frac{1}{2}\sum_{k,s=1}^{m} \frac{\partial^2}{\partial x_k \partial x_s}\big(\phi_k(t,x)\phi_s(t,x)f(t,x)\big).$$

In general case

$$\frac{\partial f(t,x)}{\partial t} = -\sum_{k=1}^{m} \frac{\partial}{\partial x_k}\big(a_k(t,x)f(t,x)\big) + \frac{1}{2}\sum_{k,s=1}^{m} \frac{\partial^2}{\partial x_k \partial x_s}\big(b_{ks}(t,x)f(t,x)\big),$$

$$a_k(t,x)(k=1,\dots,m)\;,\; b_{ks}(t,x)(k,s=1,\dots,m)$$

Find from formula

$$a_k(t,x) = \lim_{h\to 0} h^{-1}\langle x_k(t+h) - x_k(t) | X(t) = x\rangle,$$

$$b_{ks}(t,x) = \lim_{h\to 0} h^{-1}\big\langle\big(x_k(t+h) - x_k(t)\big)\big(x_s(t+h) - x_s(t)\big)|X(t) = x\big\rangle(k,s=1,\dots,m).$$

From system equation (6) we find

$$a_k(t,x) = \lim_{h\to 0} h^{-1}\big\langle s_k\sqrt{h}\phi_k(t,x) + h\phi_k(t,x)\big\rangle = \phi_k(t,x),$$

$$b_{ks}(t,x) = \lim_{h\to 0} h^{-1}\big\langle \xi_j\sqrt{h}\phi_k(t,X) + \xi_j\sqrt{h}\phi_s(t,x)\big\rangle = \phi_k(t,x)\phi_s(t,x).$$

We proved first part. And now we will find moment equations.

From (9) find

$$\int_{E_m} yy^* f\,(t+h,y,h)dy = \frac{1}{2}\int_{E_m} T\big(t,y,\sqrt{h}\big)\,T^*(t,y,h)dy + \frac{1}{2}\int_{E_m} T\big(t,y,-\sqrt{h}\big)\,T^*(t,y,h)dy,$$

or

$$D(t+h) = \frac{1}{2}\int_{E_m}\Bigg(\Big(y + h\Psi(t,y) + \sqrt{h}\Phi(t,y)\,\Big(y^* + h\Psi^*(t,y) + \sqrt{h}\Phi^*(t,y)\Big)\Big)\Bigg) +$$

$$+\Big(y + h\Psi(t,y) - \sqrt{h}\Phi(t,y)\Big)\,(y^* + h\Psi^*(t,y) - \sqrt{h}\Phi^*(t,y)f(t,y,h)dy.$$

Then

$$D(t+h) = \int_{E_m} (yy^* + hy\Psi^*(t,y) + h\Psi(t,y)yy^* + h\Phi(t,y)\Phi^*(t,y) + h^2\Psi(t,y)\Psi^*(t,y)\Big)\,f(t,y,h)dy.$$

and $\int_{E_m} yy^* f(t,y,h)dy = D(t)$, then $h \to 0$ we have equations (5).■

Lemma 1. For the system equation

$$dX(t) = AXdt + BXdw(t)$$

The system (5) transformed to system

$$\frac{dD(t)}{dt} = AD(t) + D(t)A^* + BD(t)B^*.$$

Lemma 2. For the system equation

$$dX(t) = AX(t) + BdW(t), dim\,X\,(t) = m, dim\,W\,(t) = p,$$

$$\langle dW(t)dW^*(t)\rangle = Rdt, dim\,R = p\times p,$$

The system of moment equation has form

$$\frac{dD(t)}{dt} = AD(t) + D(t)A^* + BRB^*.$$

If $Re\,\lambda_j\,(A) < 0\ (j = 1,\dots,m)$, то

$$D = \lim_{t\to\infty} D(t) = \int_0^\infty e^{At}\, BRB^* e^{A^* t} dt$$

Theorem 3. For the system equation

$$\boldsymbol{dX(t) = AX(t)dt + BX(t)dw(t),}$$

Matrix equation $\boldsymbol{D(t)} = \langle \boldsymbol{X(t)X^*(t)} \rangle$ have next form

$$\boldsymbol{\frac{dD(t)}{dt} = AD(t) + D(t)A^* + \frac{1}{2}BBD(t) + BD(t)B^* + \frac{1}{2}D(t)B^*B^*.}$$

Proof. 1 algorithm. Now constructing next equations

$$X(t+dt) = X(t) + dX(t) + \frac{1}{2}d^2X(t) + \dots =$$

$$= X(t) + \big(AX(t)dt + BX(t)dw(t)\big)$$

$$+\frac{1}{2}\Big(A\big(AX(t)dt + BX(t)dw(t)\big)dt + B\big(AX(t)dt + BX(t)dw(t)\big)dw(t)\Big) + \dots =$$

$$= X(t) + AX(t)dt + BX(t)dw(t) + \frac{1}{2}BBX(t)\big(dw(t)\big)^2 + O\left((\Delta t)^{3/2}\right).$$

Then

$$\langle X(t+dt)\rangle X^*(t+dt) = \langle X(t)X^*(t) + AX(t)X^*(t)dt +$$

$$+X(t)X^2(t)A^*dt + BX(t)X^*(t)B^*\big(dw(t)\big)^2 + \frac{1}{2}BBX(t)X^*(t)\big(dw(t1)\big)^2 +$$

$$+\frac{1}{2}X(t)X^*(t)B^*B^*\big(dw(t)\big)^2 + 0(dt)\rangle.$$

And we have matrix $D(t)$.

2 algorithm. Considering auxiliary system of equation

$$\frac{dX(t)}{dt} = AX(t) + BX(t)\xi_\lambda(t),$$

Where $\xi_\lambda(t)$ — Markov process, which has value $\pm\sqrt{\lambda}$ with probability

$$p_1(t) = P\{\omega: \xi(t) = \sqrt{\lambda}\}, p_2(t) = P\{\omega: \xi(t) = -\sqrt{\lambda}\},$$

And its probability satisfied the system equation

$$\frac{dp_1(t)}{dt} = -\lambda p_1(t) + \lambda p_2(t), \frac{dp_2(t)}{dt} = \lambda p_1(t) - \lambda p_2(t).$$

At $\lambda \to +\infty$ random process $\xi(t)$ touch to Winer process. For particular moments second order $D_1(t), D_2(t)$ we obtain the next system of differential equations

$$\frac{dD_1}{dt} = \big(A + B\sqrt{\lambda}\big)D_1 + D_1\big(A^* + B^*\sqrt{\lambda}\big) - \lambda D_1 + \lambda D_2,$$

$$\frac{dD_2}{dt} = \big(A - B\sqrt{\lambda}\big)D_2 + D_2\big(A^* - B^*\sqrt{\lambda}\big) + \lambda D_1 - \lambda D_2, D_k(0) = 0.$$

After replace

$$D(t) = D_1(t) + D_2(t), V(t) = D_1(t) - D_2(t)$$

obtain

$$\frac{dD}{dt} = AD + DA^* + \sqrt{\lambda}BV + \sqrt{\lambda}VB^*,$$

$$\frac{dD}{dt} = AV + VA^* + \sqrt{\lambda}BD + \sqrt{\lambda}DB^* - 2\lambda V.$$

Parameter λ touch to $\pm\infty$, therefore we have

$$V = \frac{1}{2\sqrt{\lambda}}(BD + DB^*) + O(\lambda^{-2}).$$

And used V, obtained at $\lambda \to +\infty$ equations (4) for matrix $D(t)$.

Comments. The formula (4) differs from the formula (1), although the same system of stochastic differential equations were considered. The fact is that different definitions for the dX(t) differential were actually used.

Theorem 4. For the system equation

$$dX(t) = \Psi\big(t, X(t)\big)dt + \Phi\big(t, X(t)\big)dw(t),$$

The system of moment equation has form

$$\frac{dM(t)}{dt} = \left\langle \Psi\big(t, X(t)\big) + \frac{1}{2}\frac{D\Phi\big(t, X(t)\big)}{Dx} \cdot \Phi\big(t, X(t)\big) \right\rangle,$$

$$\frac{dD(t)}{dt} = \Big\langle X(t)\Psi^*\big(t, X(t)\big) + \Psi\big(t, X(t)\big)X^*(t) + \Phi\big(t, X(t)\big)\Phi^*\big(t, X(t)\big) +$$

$$+\frac{1}{2}X(t)\Phi^*\big(t, X(t)\big)\frac{D\Phi^*\big(t, X(t)\big)}{Dx} + \frac{1}{2}\frac{D\Phi\big(t, X(t)\big)}{Dx}\Phi\big(t, X(t)\big)X^*(t)\Big\rangle.$$

Proof. Introduce auxiliary system of differential equation

$$\frac{dX(t)}{dt} = \Psi\big(t, X(t)\big) + \Phi\big(t, X(t)\big)\xi(t, \lambda),$$

where $\xi(t, \lambda)$ — Markov process which has two values $\pm\sqrt{\lambda}$

with probability $p_1(t) = P\{\xi(t) = \sqrt{\lambda}\}, p_2(t) = P\{\xi(t) = -\sqrt{\lambda}\}$.

Let's this probability satisfied the system equation

$$\frac{dp_1(t)}{dt} = -\lambda p_2(t) + \lambda p_2(t), \frac{dp_2(t)}{dt} = \lambda p_1(t) - \lambda p_2(t).$$

If $\lambda \to +\infty$ random process $\xi(t, \lambda)$ touch to Winer process. Now found system of equation for $f(t, x, \lambda)$ strong solution of system (18)

$$\frac{\partial f_1(t, x, \lambda)}{\partial t} = -div\left(f_1(t, x, \lambda)\left(\Psi(t, x) + \sqrt{\lambda}\Phi(t, x)\right)\right) - \lambda f_1(t, x, \lambda) + \lambda f_2(t, x, \lambda),$$

$$\frac{\partial f_2(t, x, \lambda)}{\partial t} = -div\left(f_2(t, x, \lambda)\left(\Psi(t, x) + \sqrt{\lambda}\Phi(t, x)\right)\right) + \lambda f_1(t, x, \lambda) + \lambda f_2(t, x, \lambda).$$

$$f(t, x, \lambda) = f(t, x, \lambda) + f_2(t, x, \lambda), v(t, x, \lambda) = f_1(t, x, \lambda) - f_2(t, x, \lambda).$$

For function $f(t, x\lambda), v(t, x, \lambda)$ we obtain

$$\frac{\partial f(t, x, \lambda)}{\partial t} = -\mathrm{div}\big(f(t, x, \lambda)\Psi(t, x)\big) - \sqrt{\lambda}\mathrm{div}\big(v(t, x, \lambda)\Phi(t, x)\big),$$

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$$\frac{\partial v(t,x,\lambda)}{\partial t} = -\mathrm{div}\big(v(t,x,\lambda)\Psi(t,x)\big) - \sqrt{\lambda}\mathrm{div}\big(f(t,x,\lambda)\Phi(t,x)\big) - 2\lambda v(t,x,\lambda).$$

From second equation we have

$$v(t,x,\lambda) = \frac{1}{2}div\big(f(t,x,\lambda)\Phi(t,x)\big) + O(\lambda^{-1}).$$

at $\lambda \to +\infty$

$$f(t,x) = \lim_{\lambda\to+\infty} f(t,x,\lambda), \frac{\partial f(t,x)}{\partial t} = -\mathrm{div}\big(f(t,x)\Psi(t,x)\big) + \frac{1}{2}\mathrm{div}\Big(\mathrm{div}\big(f(t,x)\Phi(t,x)\big)\Phi(t,x)\Big).$$

This is equation FPK**.**

We found the system of moment equation

$$X(t+dt) = X(t) + dX(t) + \frac{1}{2}d^2X(t) + \ldots = X(t) + \Big(\Psi\big(t,X(t)\big)dt + \Phi\big(t,X(t)\big)dw(t)\Big) +$$

$$+\frac{1}{2}\left(\frac{\partial\Psi(t,X)t)}{\partial t}dt^2 + \frac{D\Psi\big(t,X(t)\big)}{Dx}\Big(\Psi\big(t,X(t)\big)dt + \Phi\big(t,X(t)\big)dw(t)\Big)dt\right) +$$

$$+\frac{1}{2}\left(\frac{\partial\Phi\big(t,X(t)\big)}{\partial t}dt + \frac{D\Phi\big(t,X(t)\big)}{Dx}\Big(\Psi\big(t,X(t)\big)dt + \Phi\big(t,X(t)\big)dw(t)\Big)\right)dw(t) = X(t) + \Psi\big(t,X(t)\big)dt +$$

$$+\Phi\big(t,X(t)\big)dw(t) + \frac{1}{2}\times\frac{D\Phi\big(t,X(t)\big)}{Dx}\Phi\big(t,X(t)\big)\big(dw(t)\big)^2 + \ldots$$

We obtained $\mathrm{M}(t) = \langle X(t)\rangle$ and $D(t) = \langle X(t)X^*(t)\rangle$

$$\frac{dM(t)}{dt} = \left\langle\Psi\big(t,X(t)\big) + \frac{1}{2}\frac{D\Phi\big(t,X(t)\big)}{Dx}\cdot\Phi\big(t,X(t)\big)\right\rangle,$$

$$\frac{dD(t)}{dt} = \langle X(t)\Psi^*\big(t,X(t)\big) + \Psi\big(t,X(t)\big)X^*(t) + \Phi\big(t,X(t)\big)\cdot\Phi^*\big(t,X(t)\big) +$$

$$+\frac{1}{2}X(t)\Phi^*\big(t,X(t)\big)\frac{D\Phi^*\big(t,X(t)\big)}{Dx} + \frac{1}{2}\frac{D\Phi\big(t,X(t)\big)}{Dx}\Phi\big(t,X(t)\big)\cdot X^*(t)\Big\rangle..$$

DISCUSSION

On the basis of this formula, numerous problems of the theory of automatic control are solved, the theory of filtration and observing is constructed. Using this formula, they look for optimal control, minimizing dispersion, and build a linear stochastic control theory.

CONCLUSION

The moment equations for practically important classes of stochastic equations are obtained. Usually of interest is the question of the behaviours of the matrix of the second moments. solutions of the systems in question, because many optimization problems are reduced to minimization of

such a matrix solutions of the systems in question, because many optimization problems are reduced to minimization of such a matrix.

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Conflict of interests

The authors declare no conflict of interest.

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